Contact Mechanics

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Contact Mechanics: The stresses and deformations that arise when two solid bodies are brought into contact

Rubber tire on pavement

Rolling resistance:
Rubber tire on concrete is 35X steel on steel.

- stress (pressure) = force/area
- Automobile: 2600 lbs (1200 kg)
  - Front wheel load is ≈780 lbs (350 kg), rear is ≈520 lbs (235 kg)

x45

Rail/wheel: avg is about 1500X greater

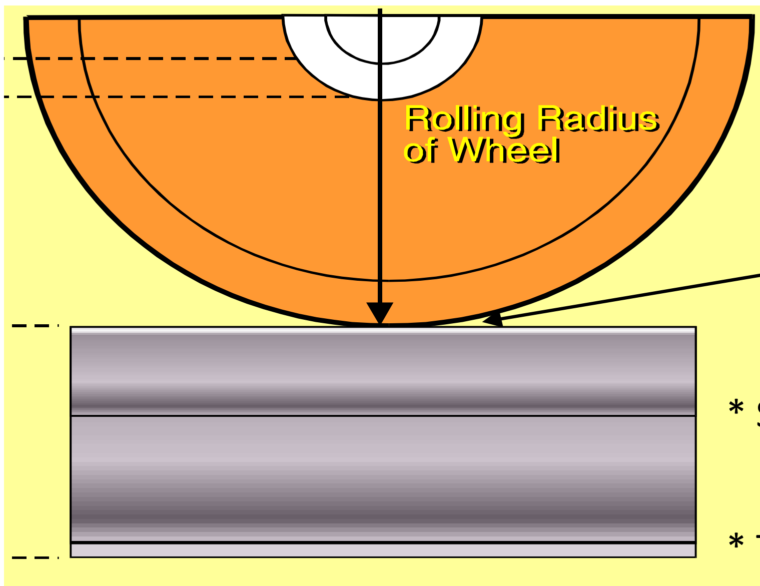
- Tire pressure: 32psi (0.22 MPa)
  - Contact stress is ≈1.6X => 50psi (0.35 MPa)
  - Contact area: 780/50=15.6 in^2 ($98 cm^2$)

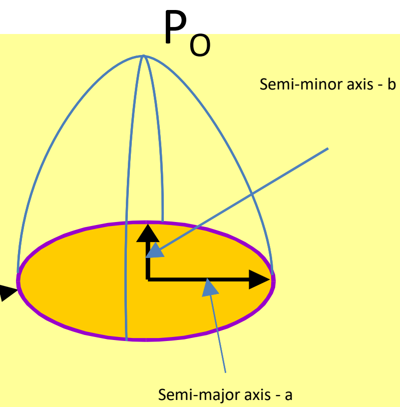
0.5 in^2, $12 mm^2$

Outline

- Hertzian contact model and stress calculations
- Pummelling
- Surface Roughness
- Creepage/slip, Creep forces
- Shakedown
- Conclusions

Hertzian contacts

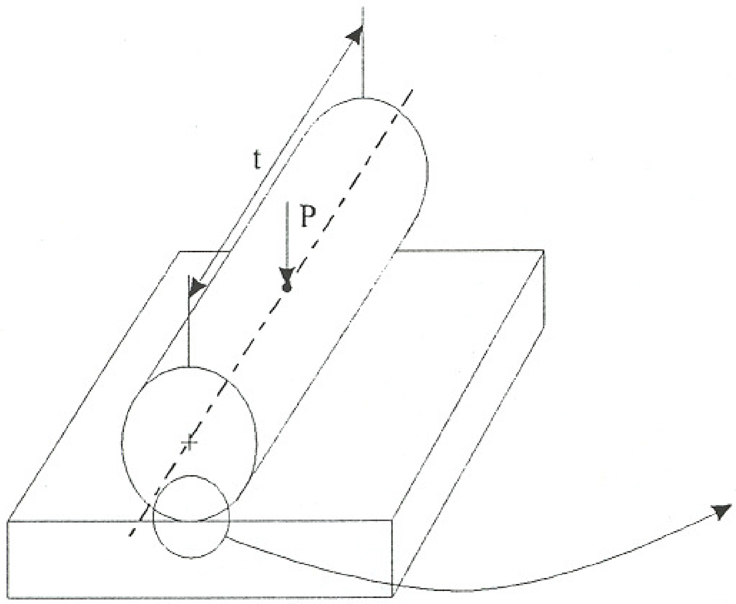




* Semi-infinite bodies
  area of contact small compared to size of the body and radii of curvature
* The surfaces are continuous and non-conforming
* Frictionless

Hertzian Line Contact

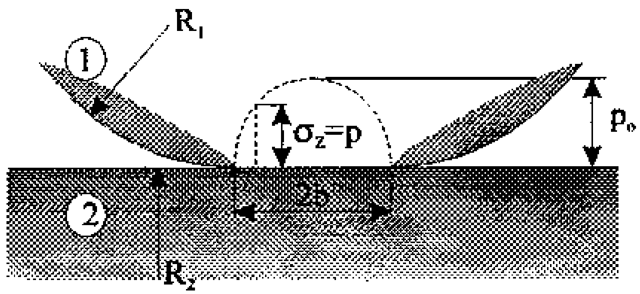
$$P_o = \left[\frac{P' E^*}{\pi R} \right]^{1/2}$$



P'=P/t = load per unit length

R = $(1/R_1+1/R_2)^{-1}$ *= effective radius*

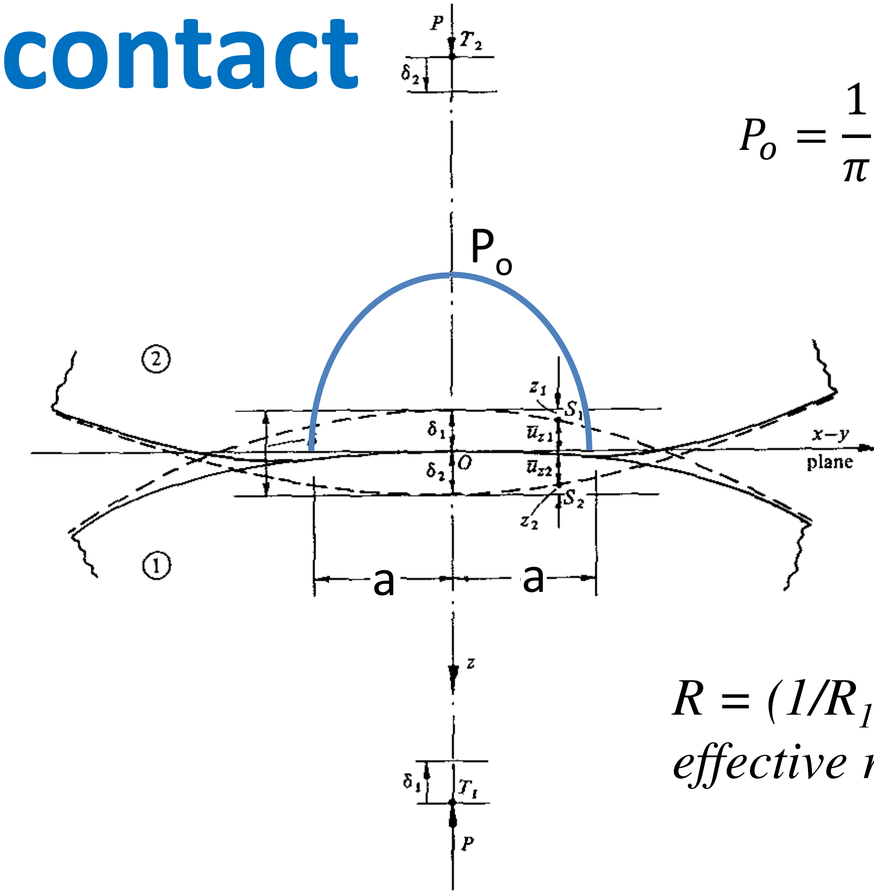
E=combined elastic modulus*



Point (circular) contact

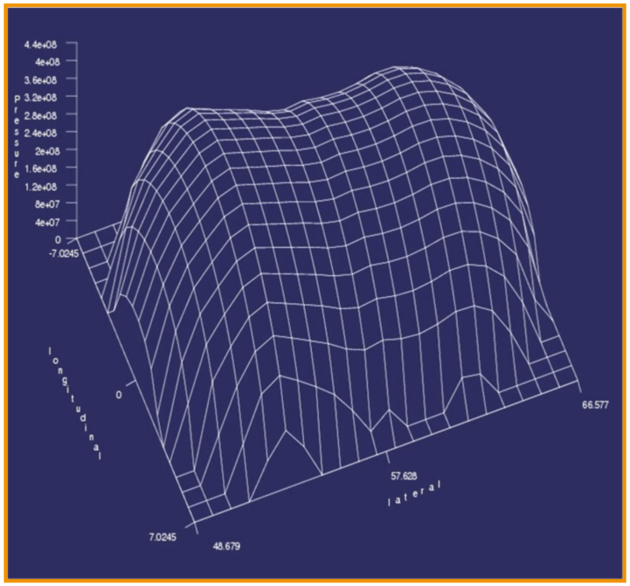
- e.g.
  - sphere on flat
  - sphere on sphere
  - two cylinders crossed at right angles

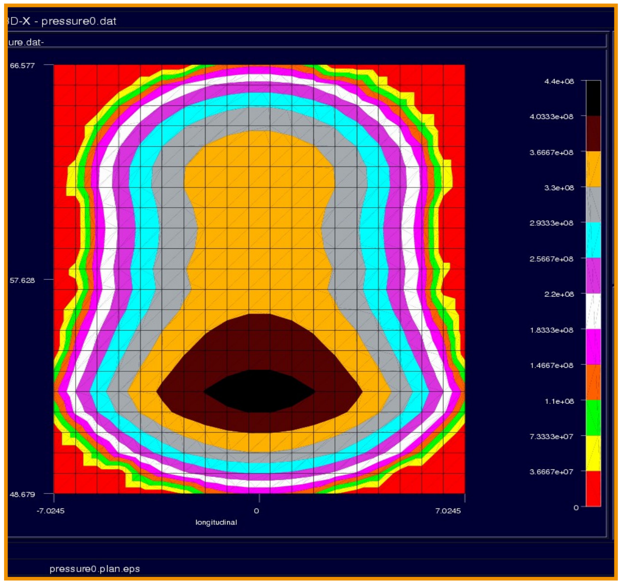
$$P_o = \frac{1}{\pi}\left[\frac{6PE^{*2}}{R^2}\right]^{1/3}$$

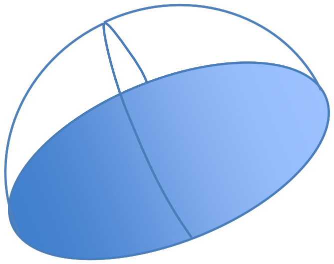


$R = (1/R_1+1/R_2)^{-1}$
effective radius

Most W/R contacts are non-Hertzian



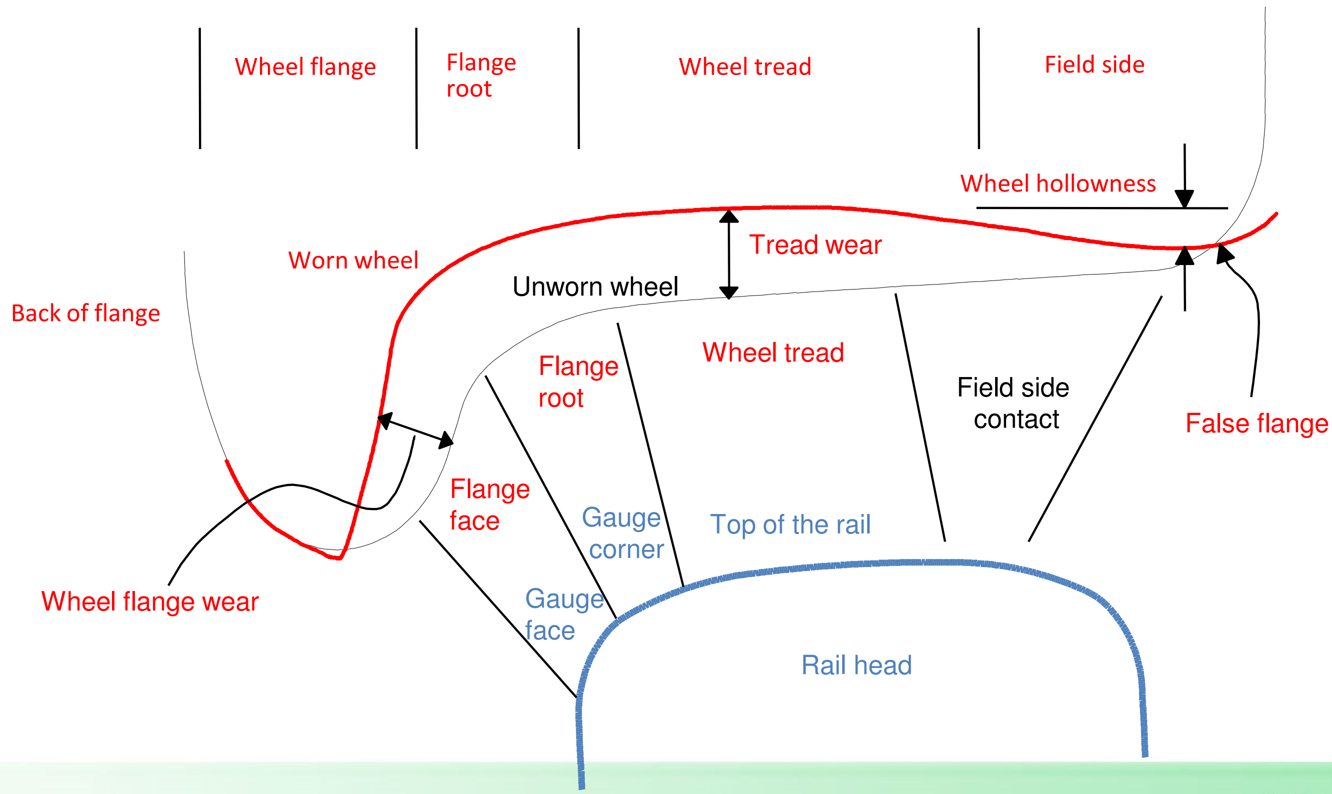




Often – elliptical is most representative

Generally: Hertzian assumption is not too bad: ±20%

Terminology



Non-Hertzian Models

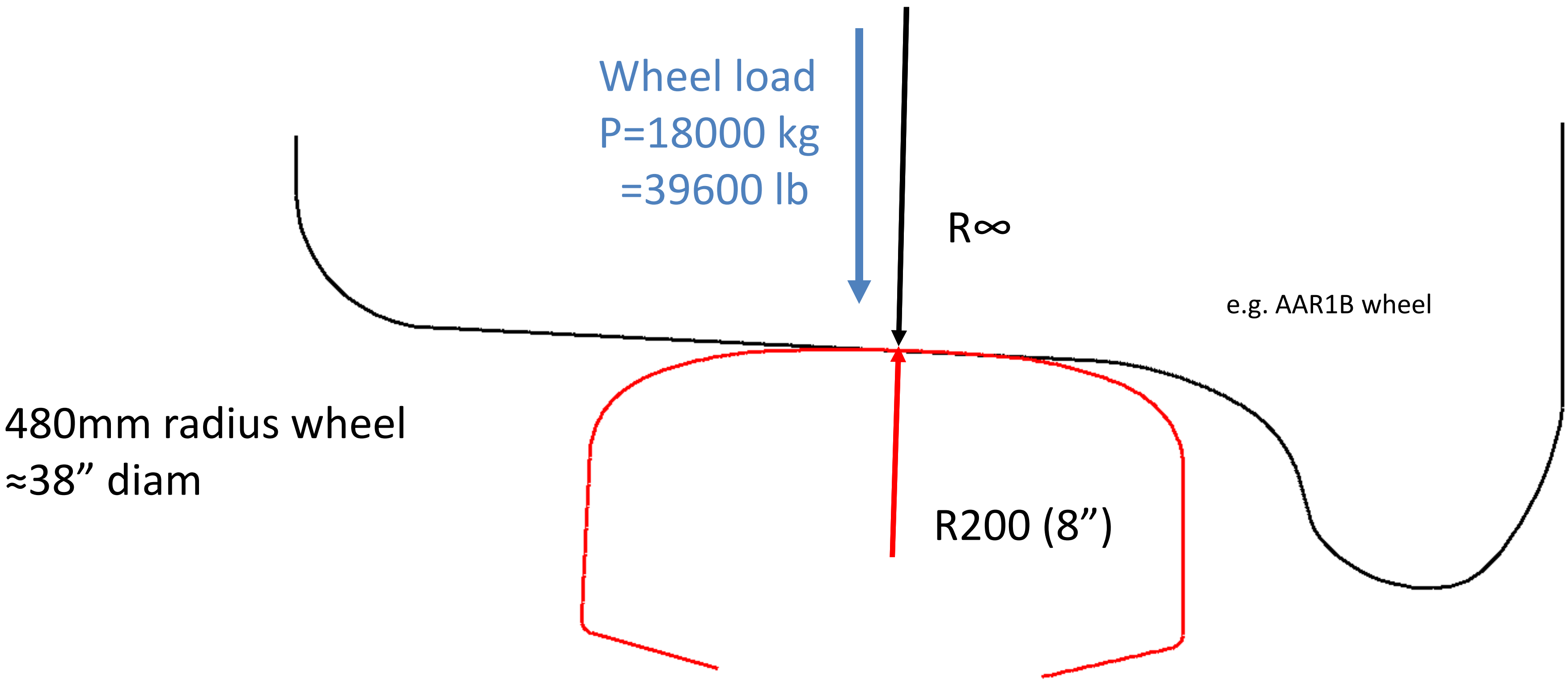
- CONTACT
- Paul and Hashemi
- FASIM
- Kik and Piotrowski
- Finite elements

Hertzian Formulae

	Line Contact Width 2b, Load P' per unit length	Circular Contact (diameter 2a, load P)
Semi-contact width or contact radius	$b = 2\left[\frac{P' R}{\pi E^*}\right]^{1/2}$	$a = \left[\frac{3}{4}\frac{PR}{E^*}\right]^{1/3}$
Maximum contact pressure ("Hertz Stress")	$P_o = \left[\frac{P' E^*}{\pi R}\right]^{1/2}$	$P_o = \frac{1}{\pi}\left[\frac{6PE^{*2}}{R^2}\right]^{1/3}$
Approach of centers	$\delta = \frac{2P'}{\pi}\left\{\frac{1-\nu_1^2}{E_1}\left[\ln\frac{4R_1}{b} - \frac{1}{2}\right] + \frac{1-\nu_2^2}{E_2}\left[\ln\frac{4R_2}{b}\right] - \frac{1}{2}\right\}$	$\delta = \frac{a^2}{R} = \frac{1}{2}\left[\frac{9}{2}\frac{P^2}{RE^{*2}}\right]^{1/3}$
Mean contact pressure	$\bar{p} = \frac{P'}{2b} = \frac{\pi}{4}P_o$	$\bar{p} = \frac{P}{\pi a^2} = \frac{2}{3}P_o$
Maximum shear stress	$\tau_{max} \cong 0.30 P_o$ at (x=0, z=0.78b)	$\tau_{max} \cong 0.31 P_o$ at (r=0, z=0.48a)

Radius (R)
Load (P)
Elastic Modulus (E)

Contact Stress Calculation #1



Contact stress calc. - TOR

- Steel wheel on Steel rail $$E^* = \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}\right)^{-1}$$

$$v_1 = v_2 = 0.29, \quad E_1 = E_2 = 200\ GPa \quad \rightarrow \quad E^* = 109\ e9\ Pa = 1.58e7\ psi$$

- 8” (200mm) rail head radius $\mathbf{R_{RT}} = 0.200\text{m}, \mathbf{R_{RL}} = \infty$
- New tapered wheel profile $\mathbf{R_{WT}} = \infty$
- Wheel radius is 480mm (≈19”) $\mathbf{R_{WL}} = 0.480\text{m}$
- Wheel load is 18000 kg X 9.81 ≈176.6 kN = **P**

Complete calculation

$$R_T = \left(\frac{1}{R_{RT}} + \frac{1}{R_{WT}}\right)^{-1} = \left(\frac{1}{0.20} + \frac{1}{\infty}\right)^{-1} = 0.20\text{m}$$

$$R_L = \left(\frac{1}{R_{RL}} + \frac{1}{R_{WL}}\right)^{-1} = \left(\frac{1}{\infty} + \frac{1}{0.480}\right)^{-1} = 0.480\text{m}$$

Elliptical contact

$$P_o = \frac{1}{\pi}\left[\frac{6PE^{*2}}{R^2}\right]^{1/3} F_1(R_L/R_T)$$

$$R = \sqrt{R_T R_L} = \sqrt{0.20 \times 0.480} = 0.31m$$

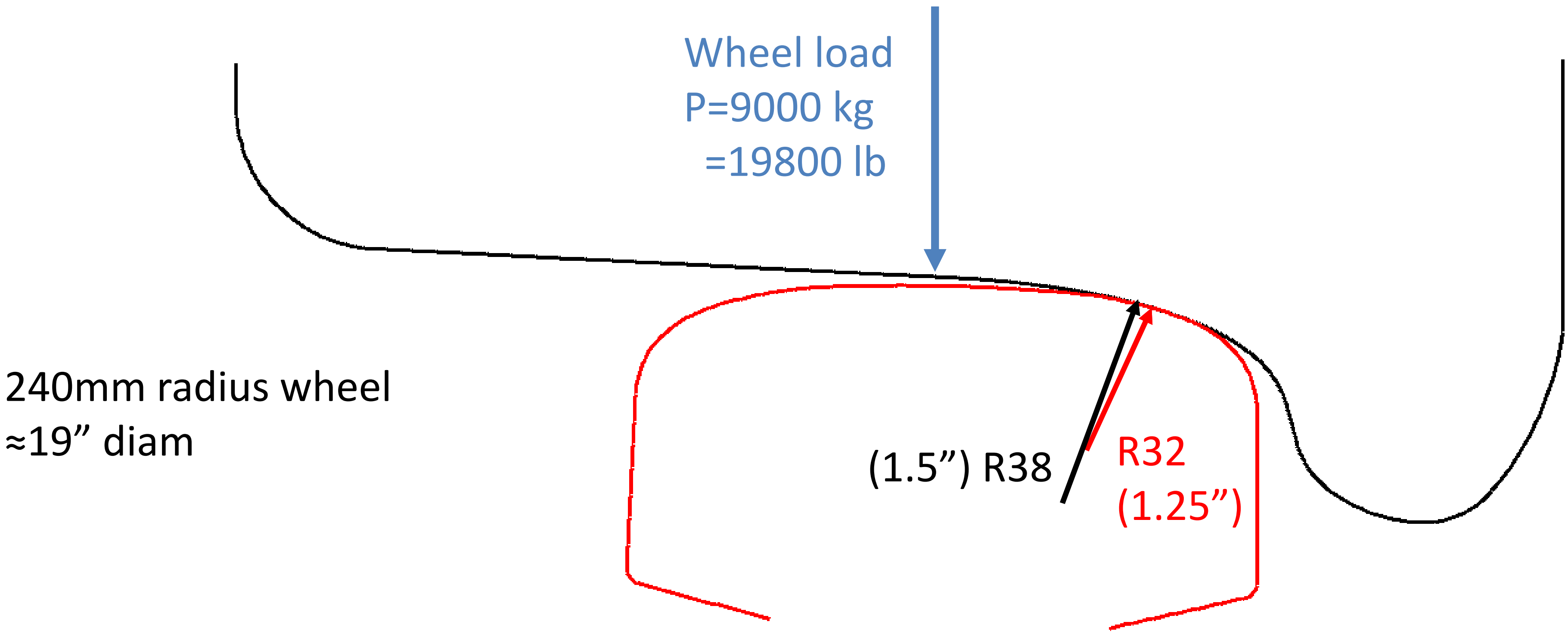
≈1.0

$$P_O = \frac{1}{\pi}\left[\frac{6 \times 176{,}600 \times (109e^9)^2}{0.31^2}\right]^{1/3} = 1616\ e^6 \text{ Pa (234ksi)}$$

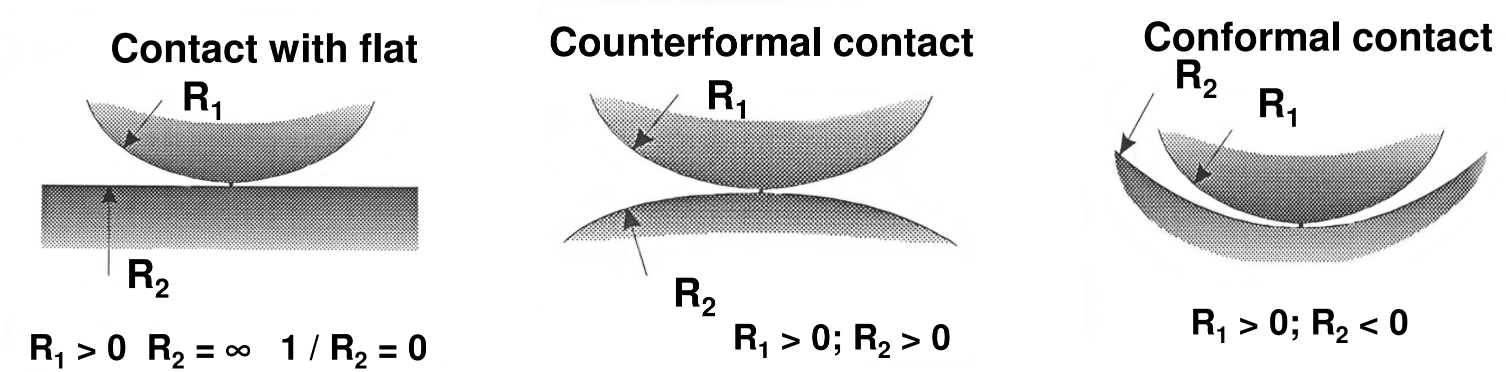
$$a = \left[\frac{3}{4}\frac{PR}{E^*}\right]^{1/3} F_1(R_L/R_T)$$

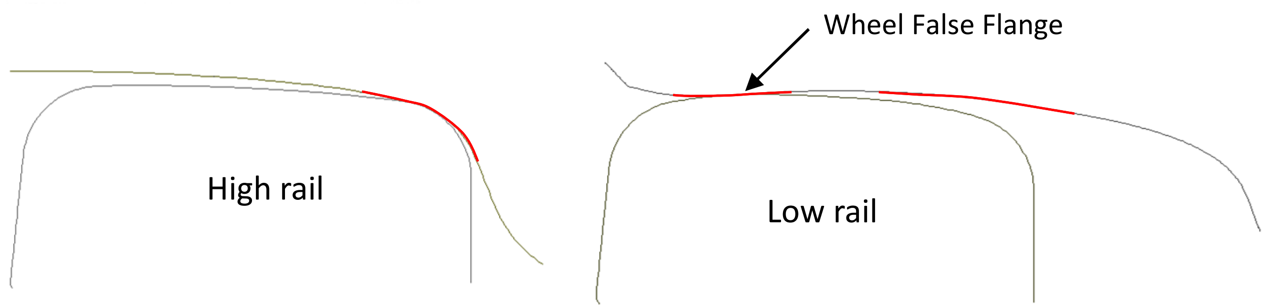
$$a = \left[\frac{3}{4}\ \frac{176600 \times 0.31}{109e^9}\right]^{1/3} = 0.00722\text{ m} \equiv 7.22\text{mm} \rightarrow 14.5\text{mm diam, 9/16''}$$

Contact Stress calc. – gauge shoulder



Hertzian Contacts – Sign Convention





Contact stress calc. – rail shoulder

- 32 mm radius $\mathbf{R_{RT}} = 0.032\text{m}, \mathbf{R_{RL}} = \infty$
- 38 mm flange root radius $\mathbf{R_{WT}} = -0.038\text{m}$
- Wheel radius is 240 mm $\mathbf{R_{WL}} = 0.240\text{m}$

$$\mathbf{R_L} = 0.240\text{m}$$

$$\mathbf{R_T} = \left(\frac{1}{0.032} - \frac{1}{0.038}\right)^{-1} = 0.2027$$

$$R = \sqrt{0.240 \times 0.2027} = 0.221$$

Complete calculation

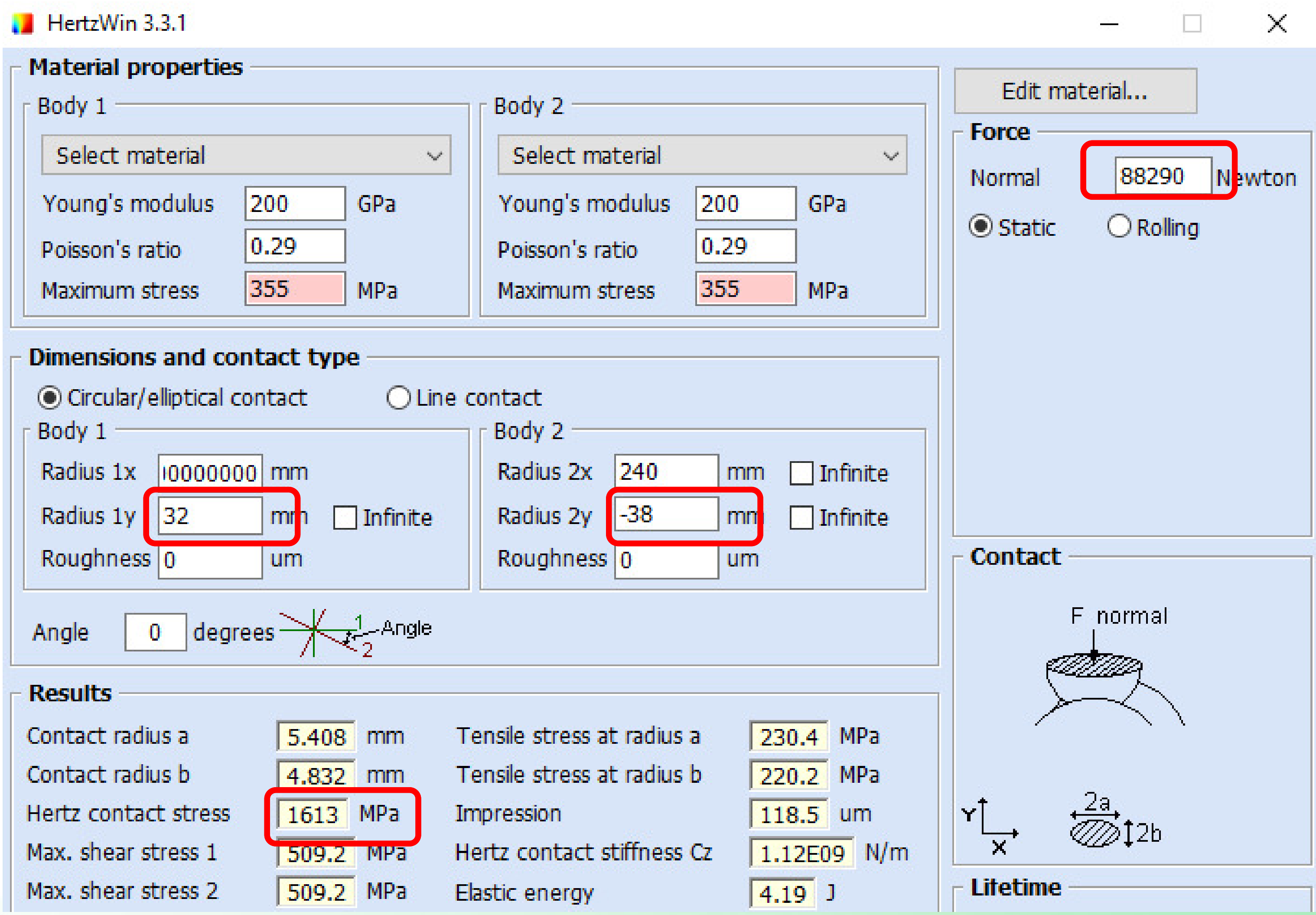
$$P = 9000 \times 9.81 = 88290N$$

$$P_o = \frac{1}{\pi}\left[\frac{6PE^{*2}}{R^2}\right]^{1/3}$$

$$P_O = \frac{1}{\pi}\left[\frac{6 \times 88290 \times \left(109e^9\right)^2}{0.221^2}\right]^{1/3} = 1608\ e^6 \text{ Pa}$$

$$a = \left[\frac{3}{4}\frac{PR}{E^*}\right]^{1/3}$$

$$a = \left[\frac{3}{4}\frac{88290 \times 0.221}{109e^9}\right]^{1/3}$$ = 0.00512 m ≡ 5.12mm →10.24mm diam, 13/32


HertzWin 3.3.1
Material properties
Body 1
Select material
Young's modulus 200 GPa
Poisson's ratio 0.29
Maximum stress 355 MPa
Body 2
Select material
Young's modulus 200 GPa
Poisson's ratio 0.29
Maximum stress 355 MPa
Edit material...
Force
Normal 88290 Newton
Static
Rolling
Dimensions and contact type
Circular/elliptical contact
Line contact
Body 1
Radius 1x 10000000 mm
Radius 1y 32 mm Infinite
Roughness 0 um
Body 2
Radius 2x 240 mm Infinite
Radius 2y -38 mm Infinite
Roughness 0 um
Angle 0 degrees
Angle
Contact
F normal
2a
2b
Y
X
Results
Contact radius a 5.408 mm
Contact radius b 4.832 mm
Hertz contact stress 1613 MPa
Max. shear stress 1 509.2 MPa
Max. shear stress 2 509.2 MPa
Tensile stress at radius a 230.4 MPa
Tensile stress at radius b 220.2 MPa
Impression 118.5 um
Hertz contact stiffness Cz 1.12E09 N/m
Elastic energy 4.19 J
Lifetime

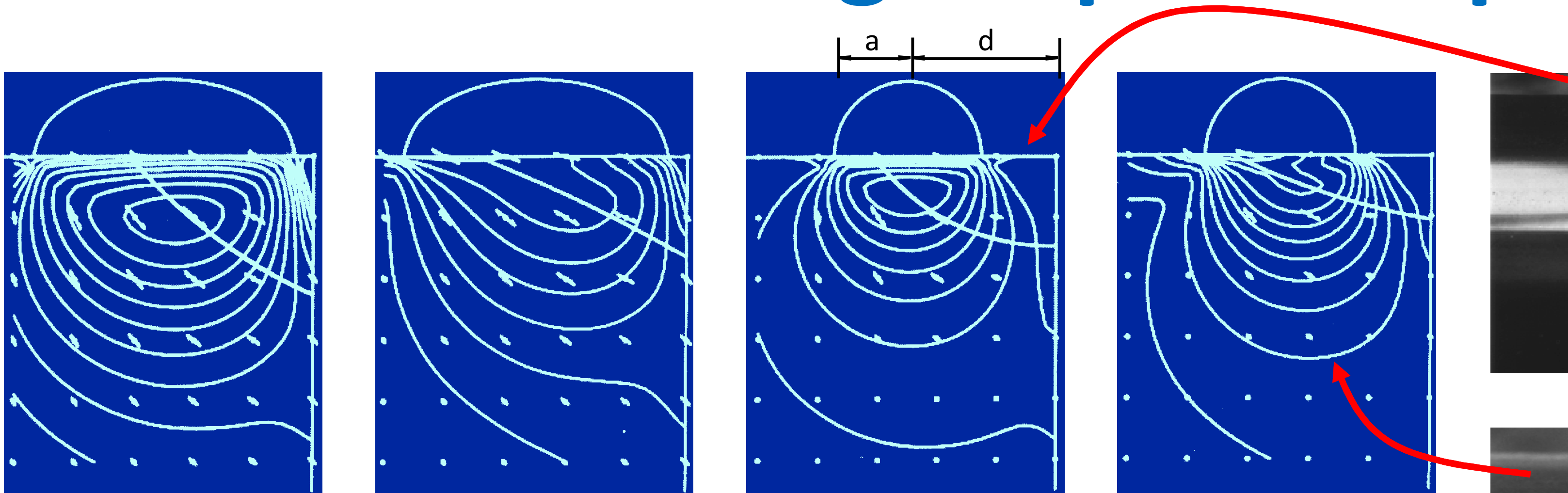
Rail/Wheel: Hertzian Contact Stress (MPa)

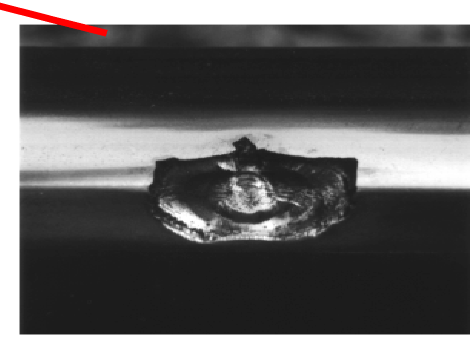
$$P_o = \left(\frac{6PE^{*2}}{\pi^3 R_e^2} \right)^{1/3} \times \left[\; F_1 (R_L / R_T)^{-2/3} \; \right]$$

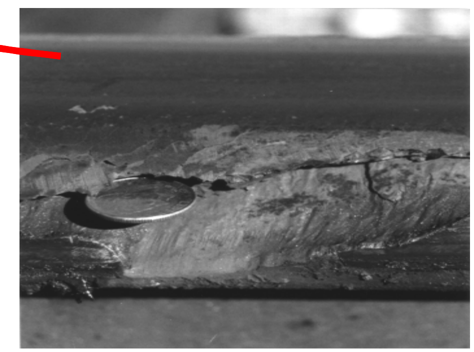
spherical contacts — accounts for ellipticity

Location	Traverse Radius Rail (mm)	Traverse Radius Wheel (mm)	Load, Wheel Radius: 18 Tonnes 480 mm		18 Tonnes 240 mm		9 Tonnes 240 mm	
Rail Crown	+200	-300	1130	(1.00)	1438	(1.27)	1141	(1.01)
	+75	-100	1428	(1.26)	1794	(1.59)	1424	(1.26)
	+100	-300	1819	(1.61)	2267	(2.01)	1800	(1.59)
	+200	infinity	1645	(1.46)	2053	(1.82)	1629	(1.44)
Rail Shoulder	+32	-38	1637	(1.45)	2043	(1.81)	1622	(1.44)
	+32	-44	1984	(1.76)	2469	(2.18)	1960	(1.73)
Flange Root	+8	-9.5	2678	(2.37)	3317	(2.94)	2632	(2.33)
False Flange	+300	+50	2845	(2.52)	3520	(3.12)	2794	(2.47)

Elastic loading of quarter space





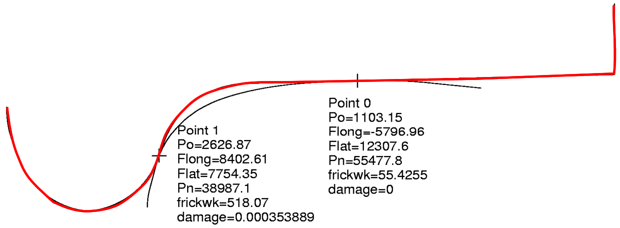


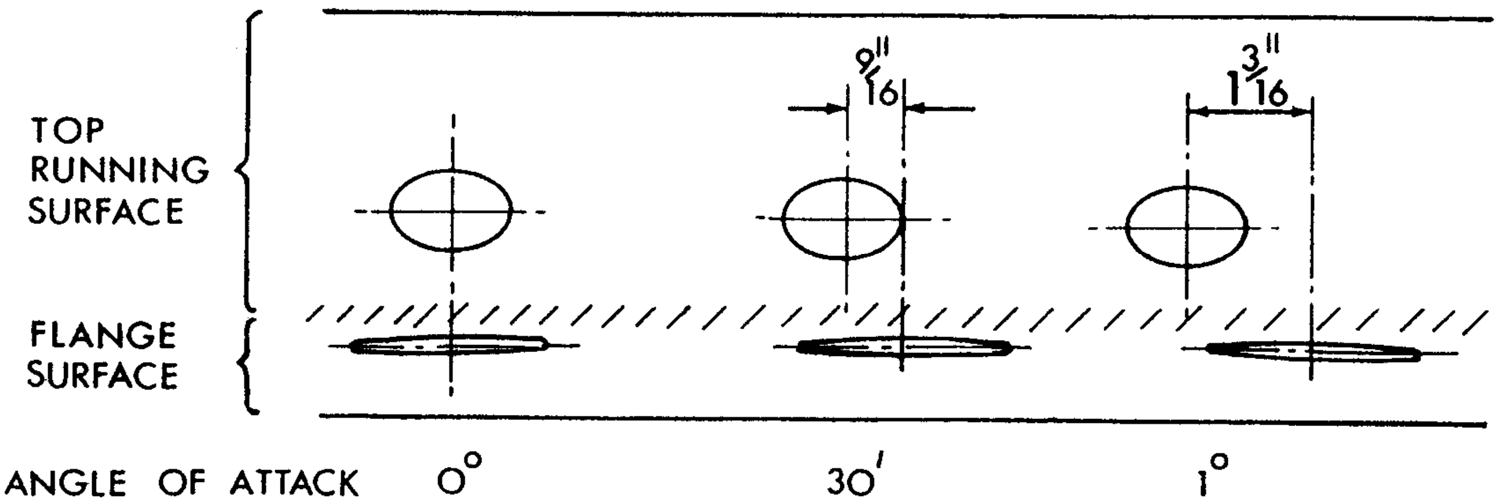
d/a=1.14 T/N=0.0	d/a=1.14 T/N=0.4	d/a=2.0 T/N=0.0	d/a=2.0 T/N=0.4

- **Leads to gauge corner collapse**
- **In lubricated track, the DSS's are "cylindrical"**
- **In dry track, the DSS's are "flatter" or "straight"**
- **Collapse is stronger if closer to the edge**

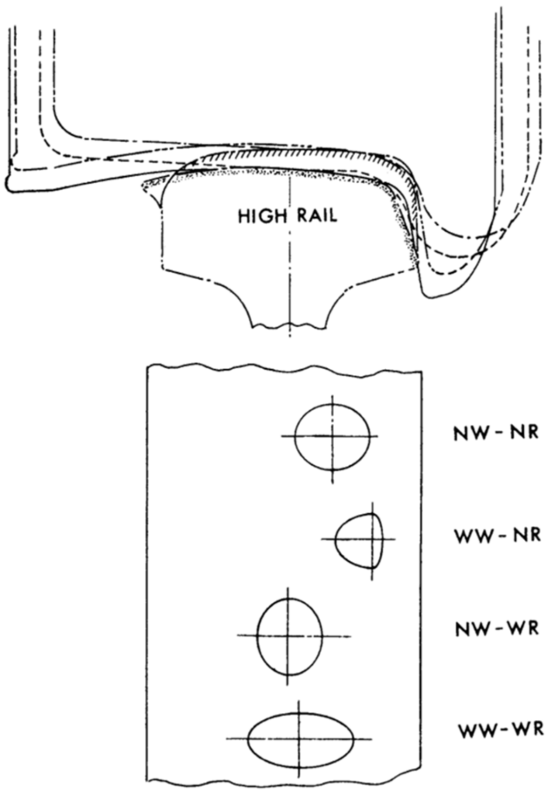
Wheel/rail contact

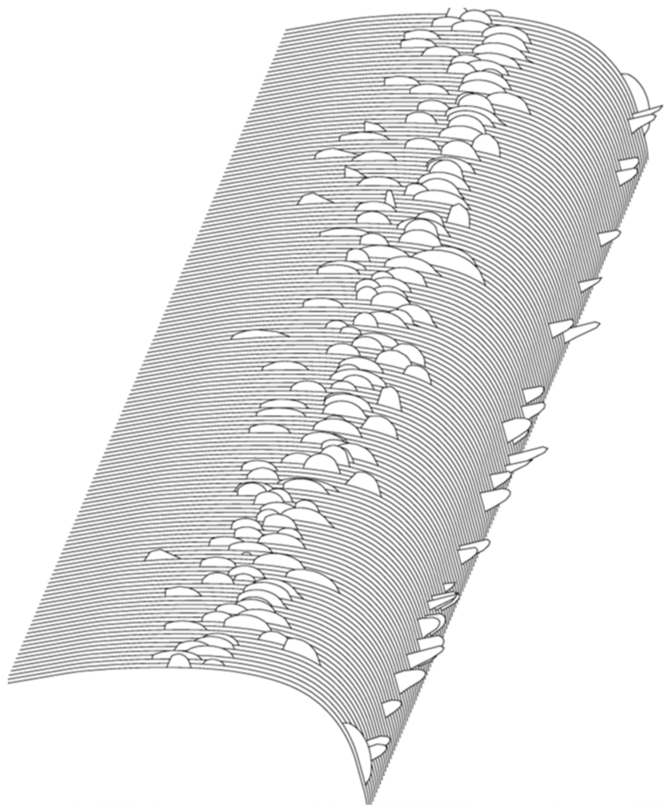
- Plan view of contact ellipses on high rail for different angles of attack

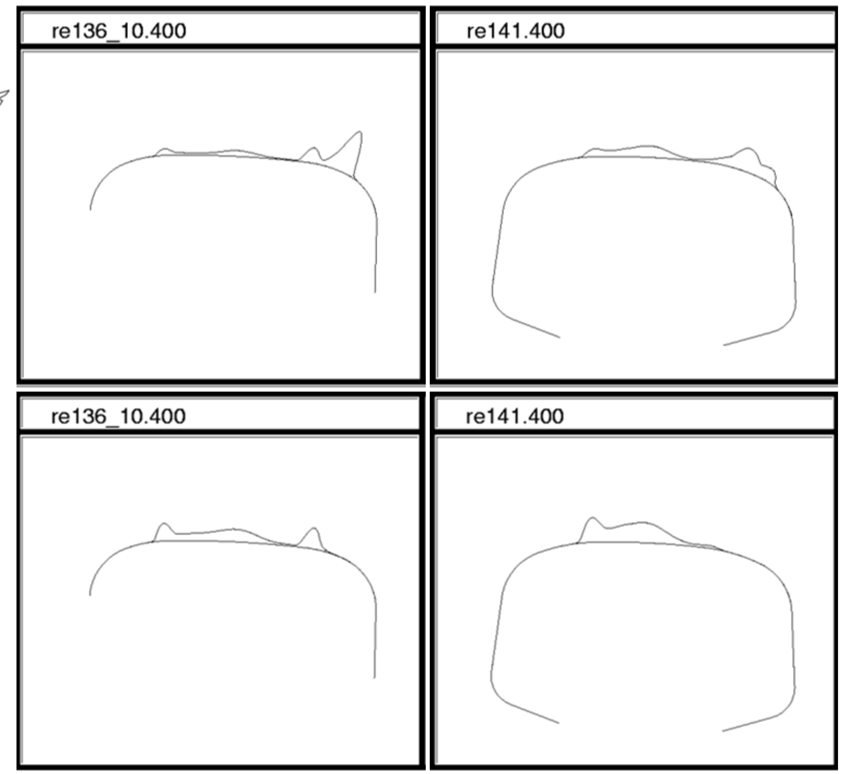




Pummelling


HIGH RAIL
NW - NR
WW - NR
NW - WR
WW - WR

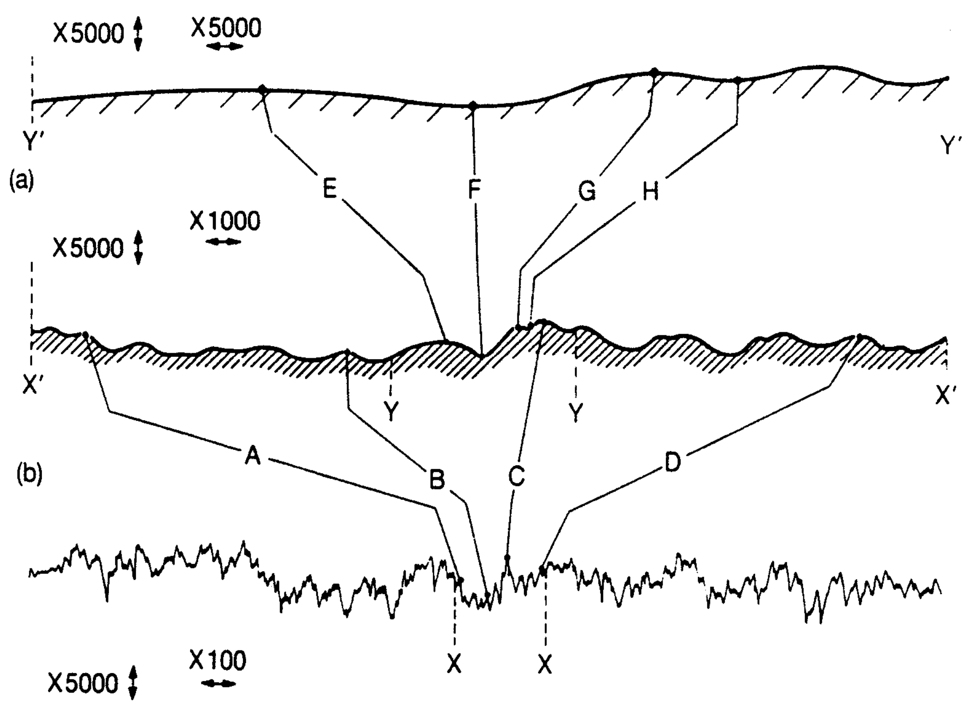



re136_10.400
re141.400
re136_10.400
re141.400

The influence of

SURFACE ROUGHNESS

Surface Roughness



On a micro-scale, all surfaces are rough

from Dagnall H, *Exploring Surface Texture*, Rank Taylor Hobson (1980).

Contact between real surfaces

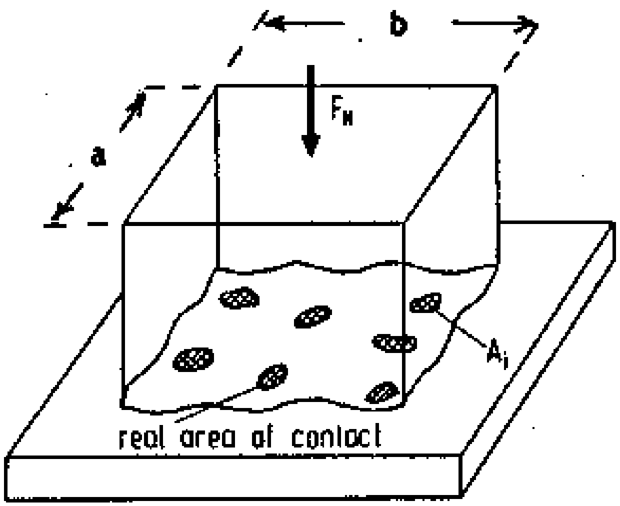
- Real area of contact is much smaller than the nominal area
- Apparent area:

$$A_A = ab$$

- Real area

$$A_R = \sum_{i=1}^{n} A_i$$

- Pressure = load/area



Contact Stress

- Elastic contact models can be applied with errors of only a few percent if the combined roughness of the two surfaces is less than about 5% of the bulk elastic compression, i.e.

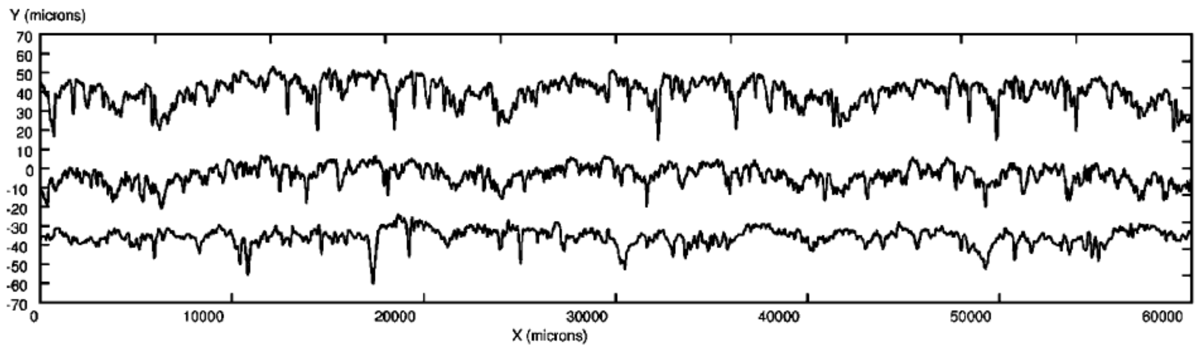
$$\alpha \equiv \frac{\sigma}{\delta} = \sigma\left(\frac{16E^{*}R}{9P}\right)^{1/3} < 0.05$$

KL Johnson, Contact Mechanics Section 13.5

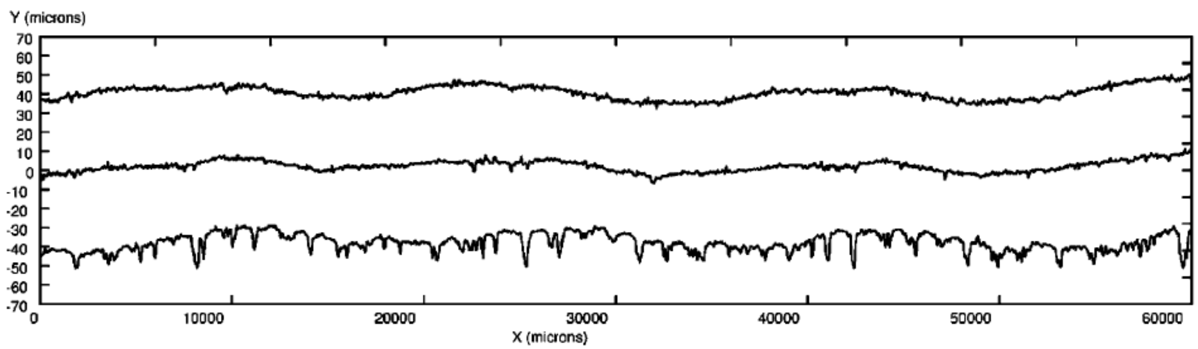
- Hertzian spring: 0.05 – 0.15mm => 2.5 – 7.5 μm

Roughness from rail grinding

US Transit

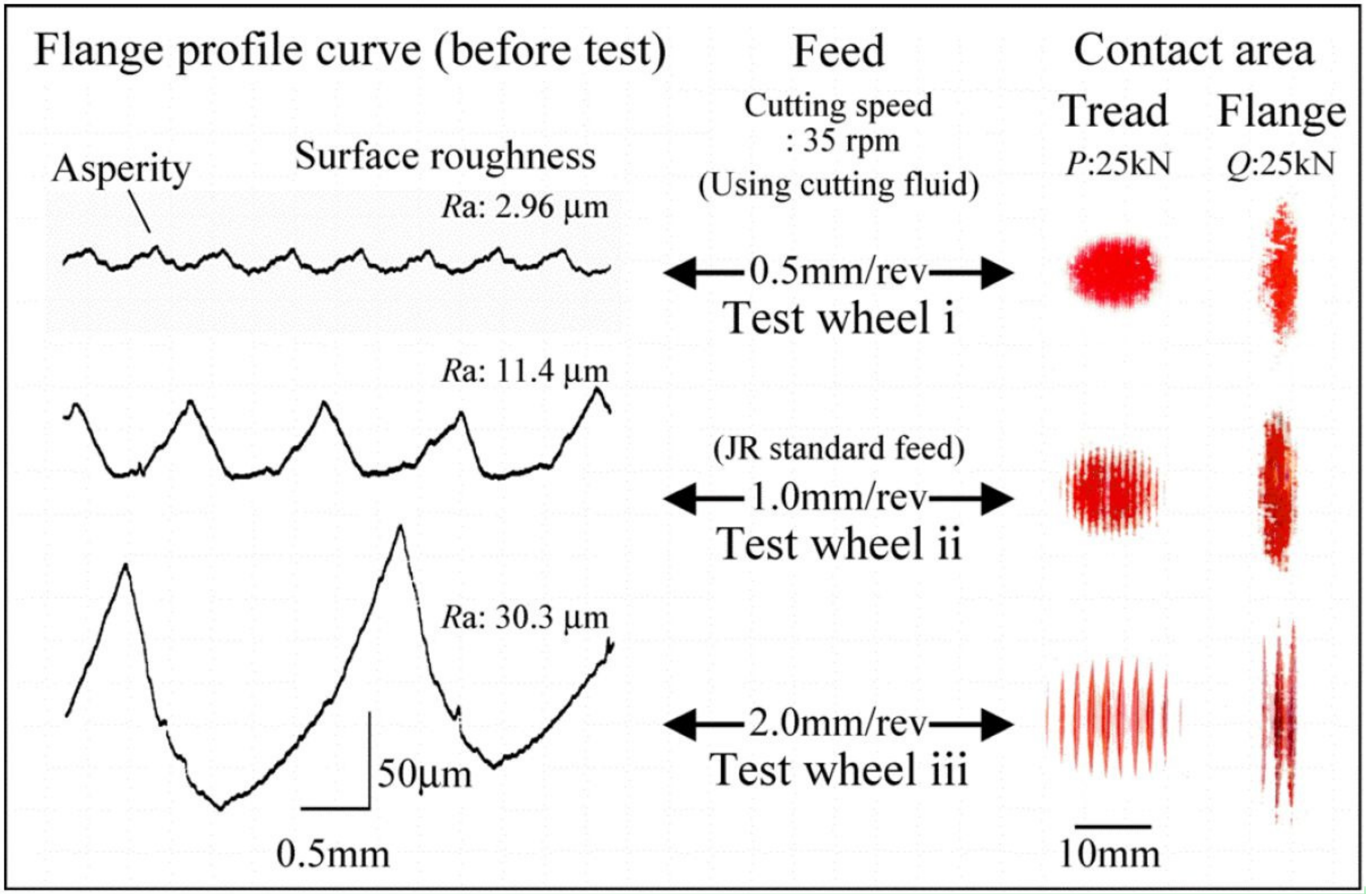


(A) Immediately after grinding
(σ is typically about 20 microns)



(B) Approx. 2MGT after grinding
(σ is typically <1 micron)

The rough wheel and wheel climb



T. Ban et al, *A study on the coefficient of friction between rail gauge corner and wheel flange focussing on wheel machining*, Proceedings International Wheelset Congress, Orlando, 2004

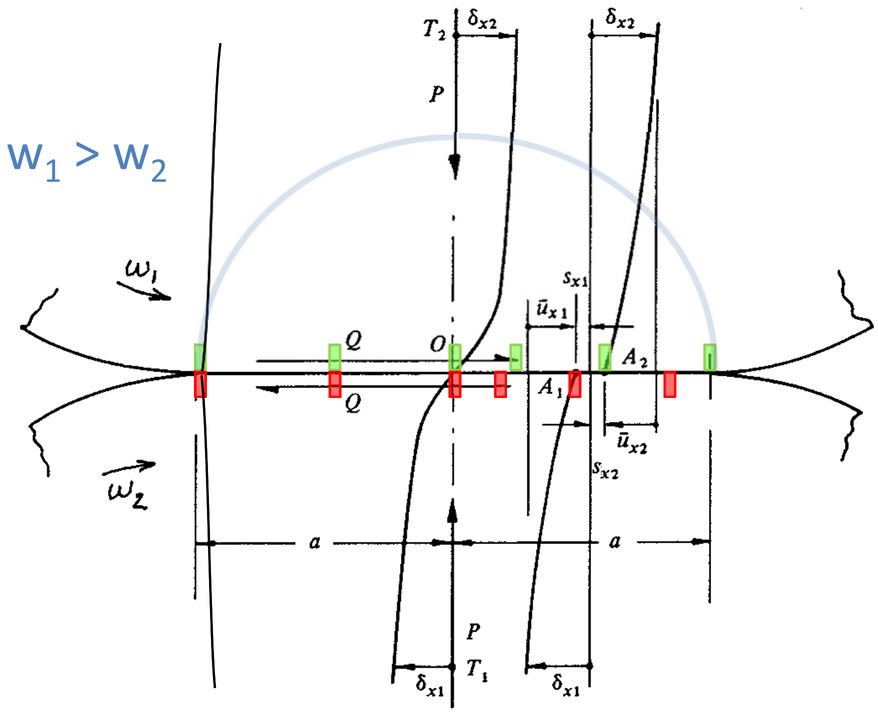
Surface Roughness - conclusion

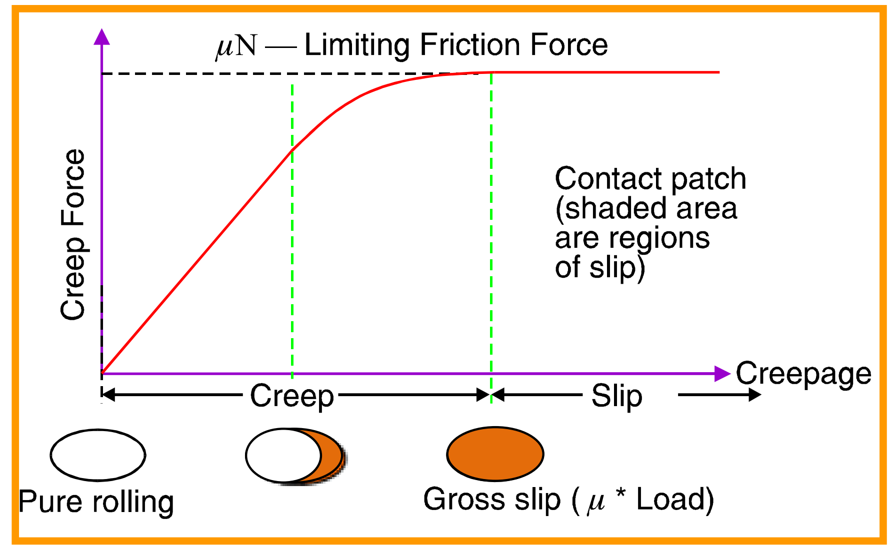
- Important
  - high frequency phenomena (noise, vibration)
  - Deformation of the micro-surface layer
- Little impact
  - bulk contact stresses
  - Wheel/rail forces
- Wheel roughness ⟷ wheel climb ??

CREEPAGE/SLIP

Stick and Slip in the Contact Patch

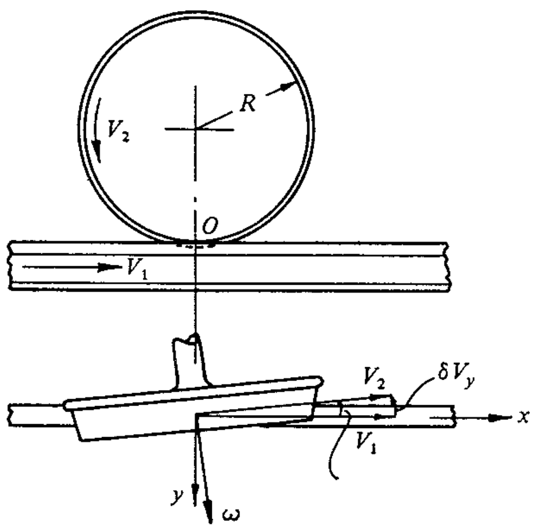
From KL Johnson (1987), Contact Mechanics, Fig 7.6

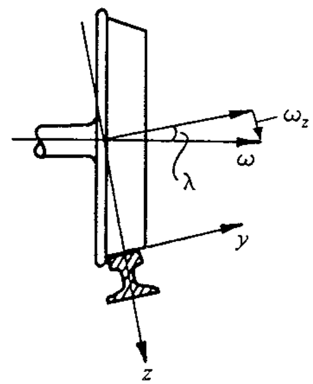




Elastic deformation in rolling bodies in stick and slip regions in rolling sliding contact

Creepage in a single wheel/rail contact





Longitudinal Creepage

$$\psi_X = \frac{V_2 - V_1}{V_1}$$

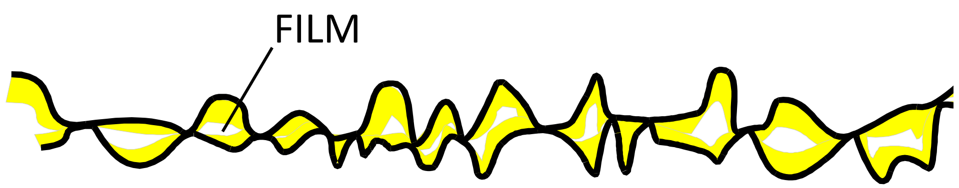
Lateral Creepage

$$\psi_Y = \frac{\delta V_Y}{V_1} = \tan \gamma$$

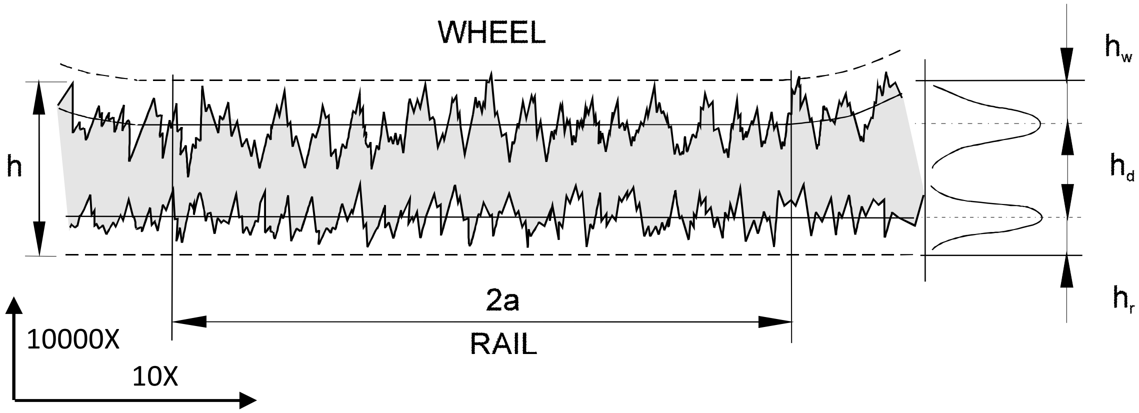
Spin Parameter

$$\Phi = \omega \frac{(ab)^{1/2}}{V_1 R} = \left(\frac{(ab)^{1/2}}{R} \right) \tan \lambda$$

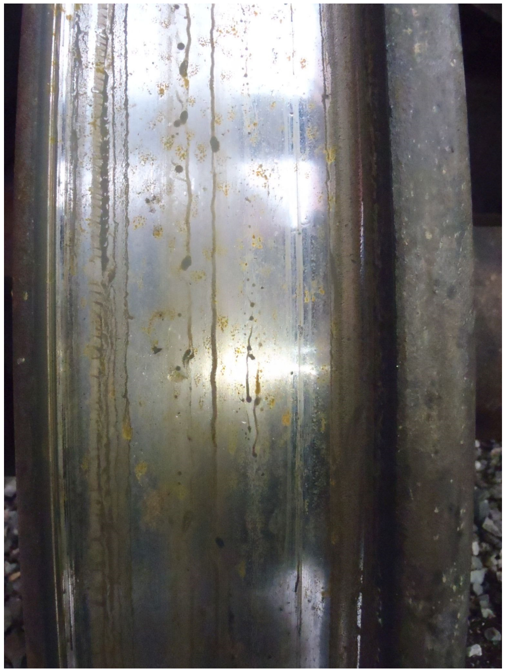
Third-body layer



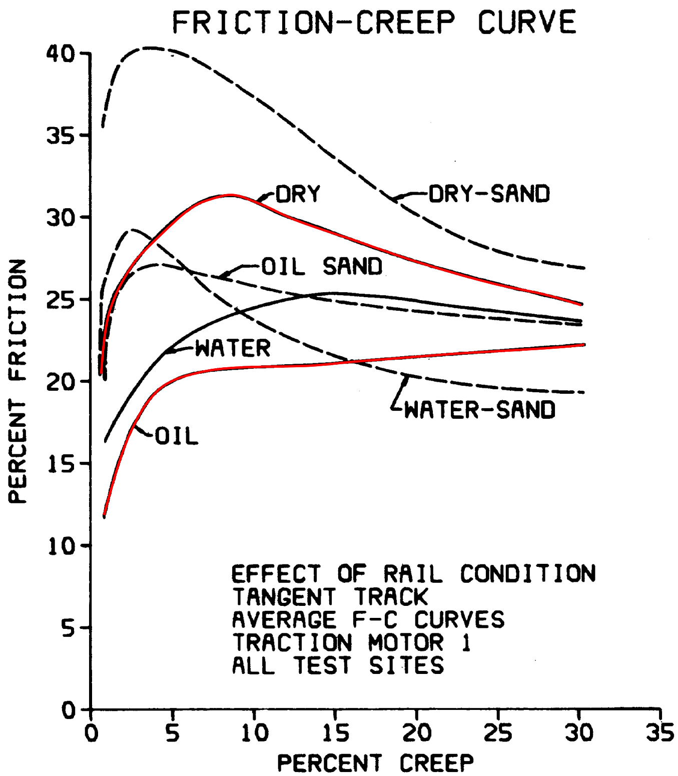
- Petrochemical: oil, soap, grease
- Solid / mechanical: moly, graphite
- Chemical: phosphate, salts, etc.

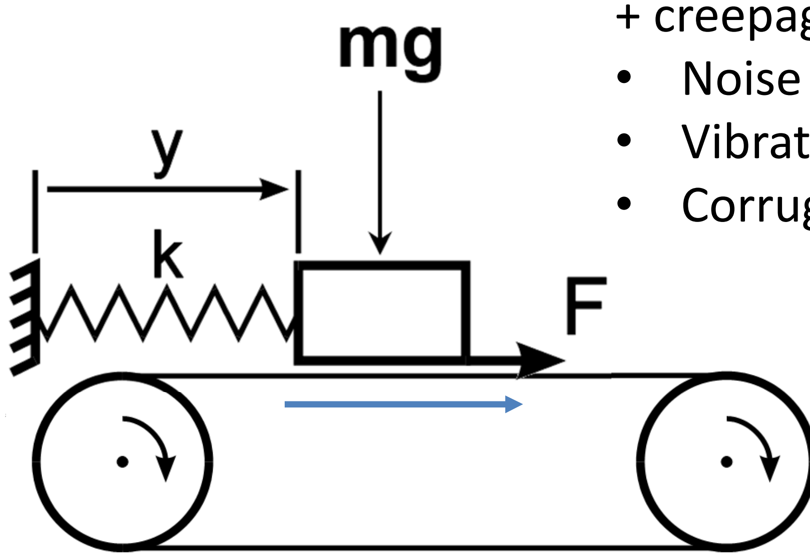


LAYERS:
Any microscopic mixture of solid and semi-solid particles



Stick-Slip – Negative Friction



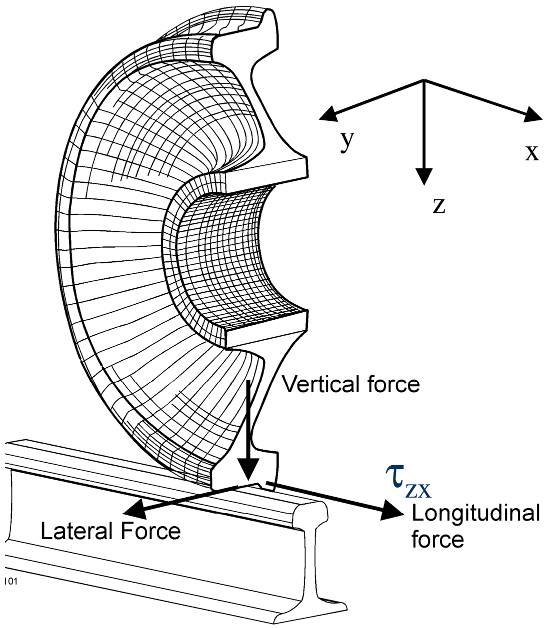


Negative Friction Characteristic + creepage

- Noise
- Vibration
- Corrugation

Wheel/rail stresses

- Vertical, longitudinal, and lateral forces
- Lead to a complex stress field
  - Compressive, tensile and shear stress components
- P_0 is maximum normal contact stress
- Important stresses = τ_{zx}, τ_{zy}
  - The stress on the z plane in the x and y direction
  - Cause shear of rail surface



Effect of shear stress

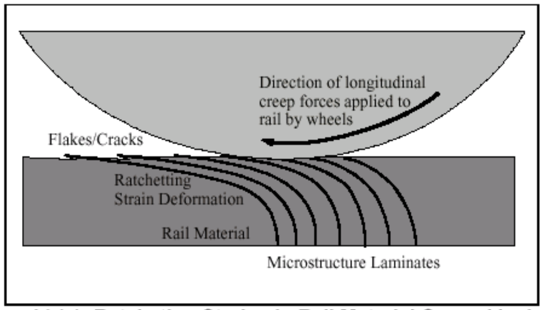
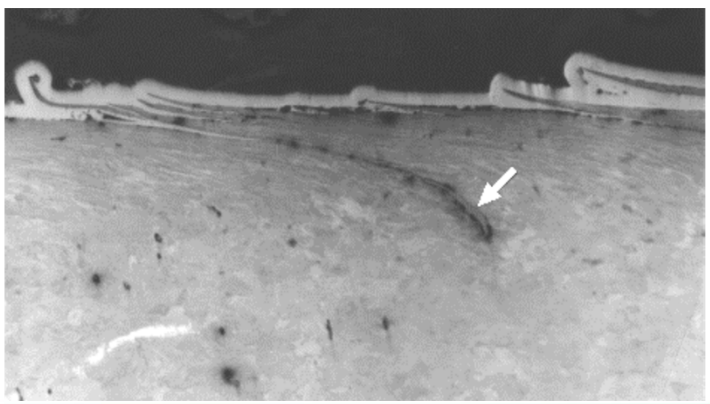
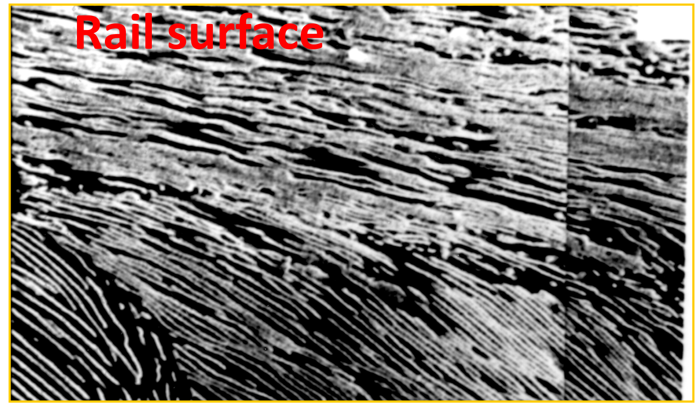
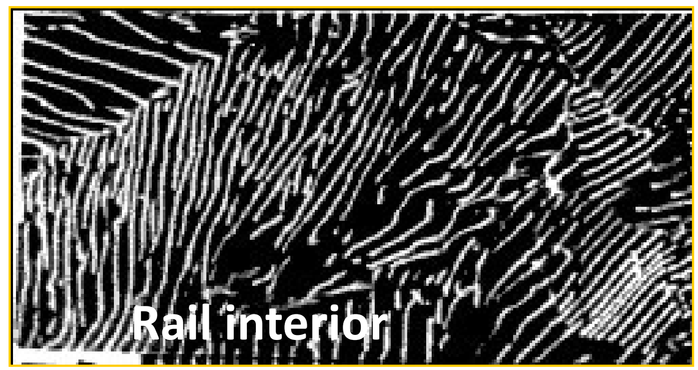


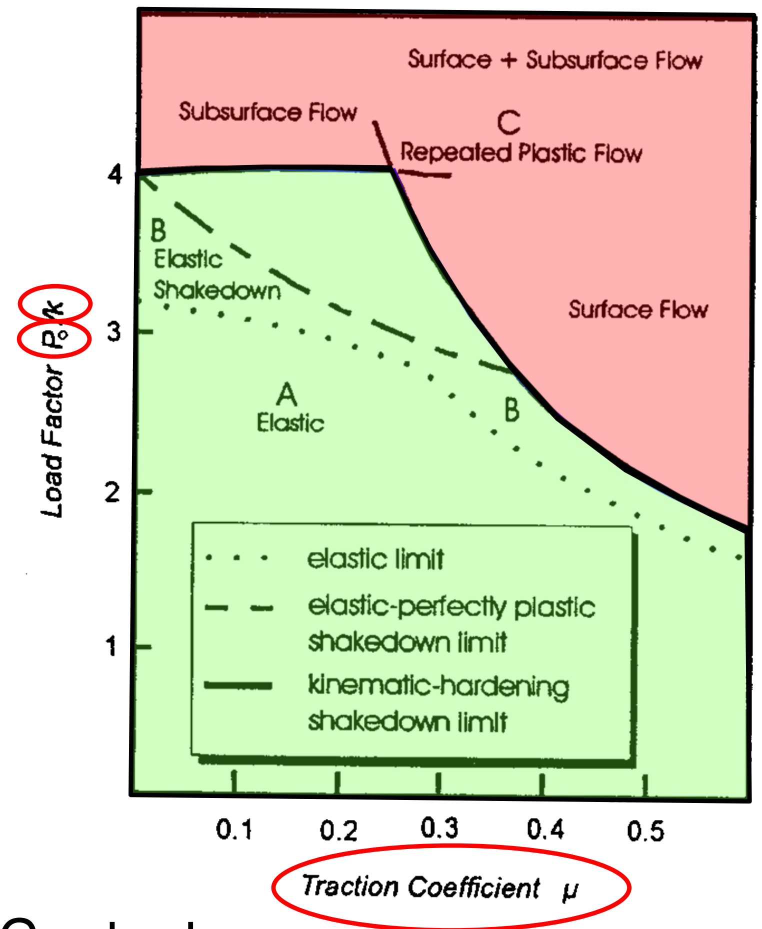
Figure 14.(c): Ratcheting Strains in Rail Material Caused by Large Longitudinal Creep Forces Between Wheel and Rail



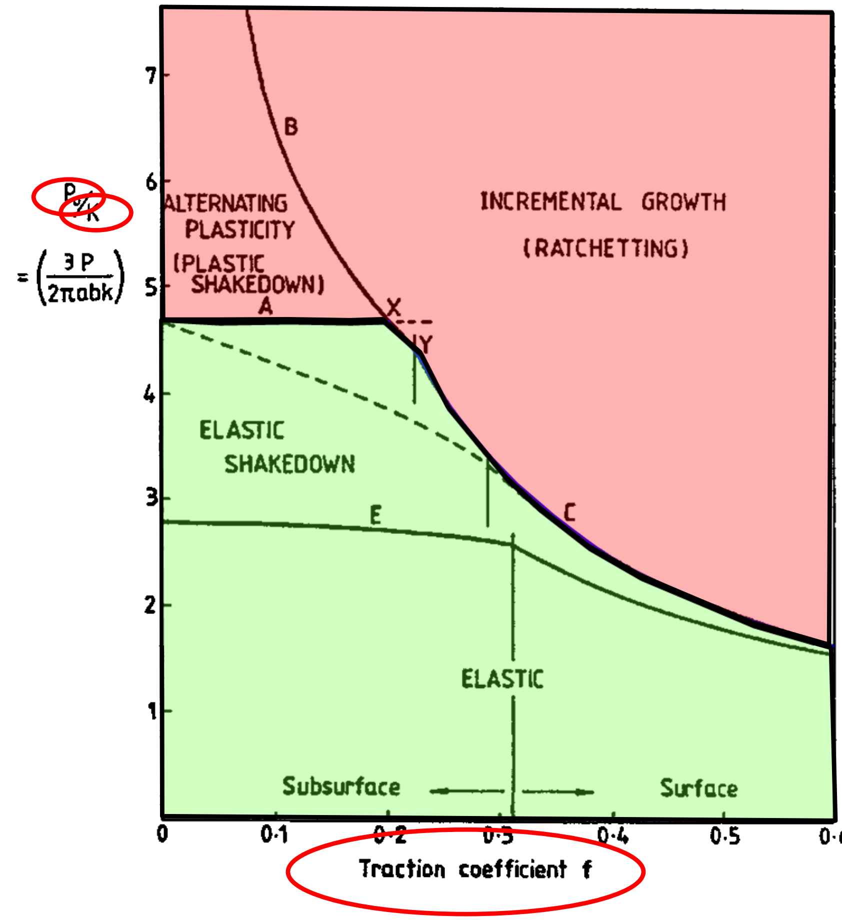




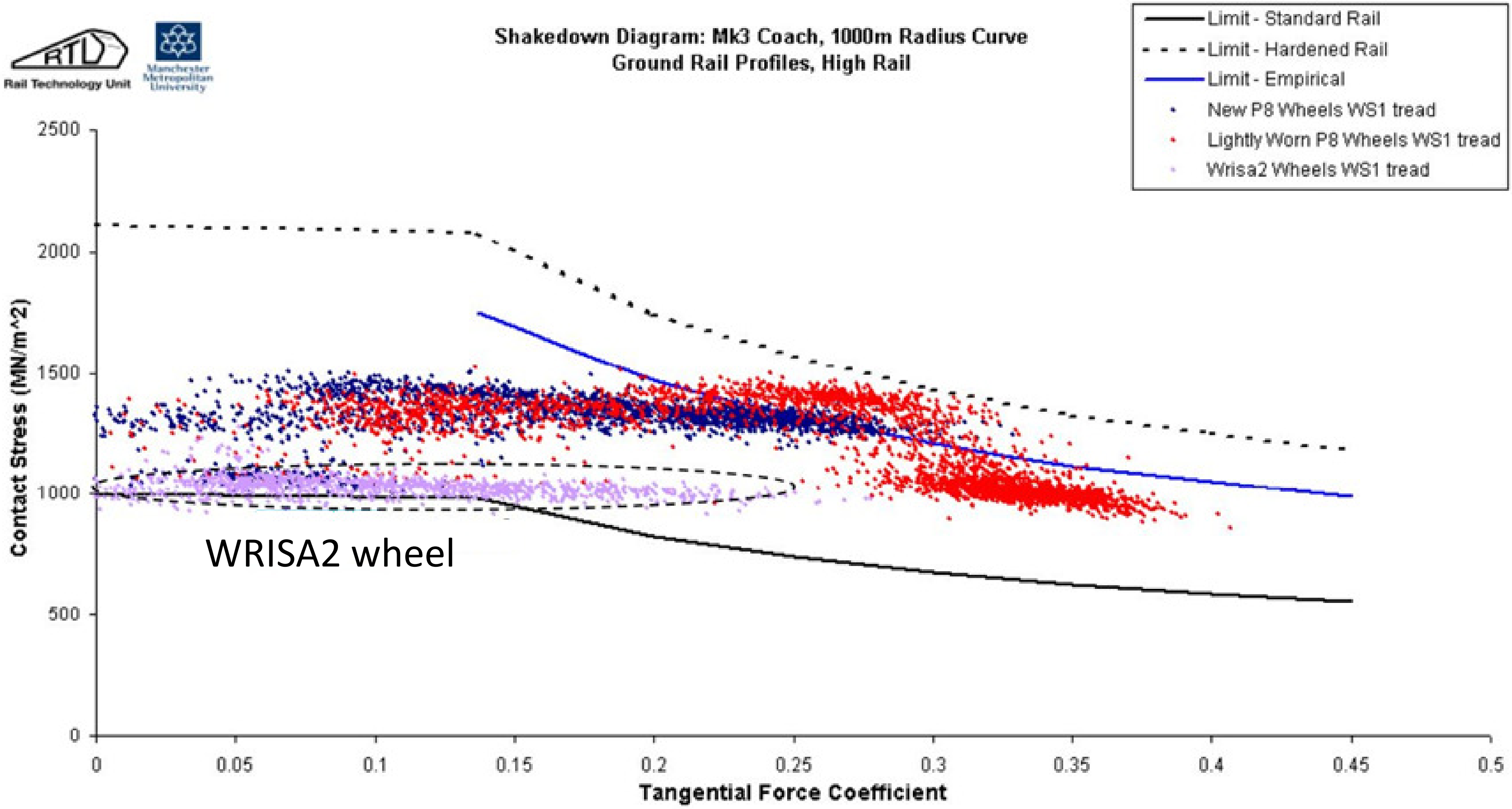
SHAKEDOWN



Line Contacts



Point (circular) Contacts


Rail Technology Unit
Manchester Metropolitan University
Shakedown Diagram: Mk3 Coach, 1000m Radius Curve
Ground Rail Profiles, High Rail
Limit - Standard Rail
Limit - Hardened Rail
Limit - Empirical
New P8 Wheels WS1 tread
Lightly Worn P8 Wheels WS1 tread
Wrisa2 Wheels WS1 tread
Contact Stress (MN/m^2)
2500
2000
1500
1000
500
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Tangential Force Coefficient
WRISA2 wheel

Conclusions

- **Hertzian** contacts
  - Linear elasticity - line, point and elliptical contacts
    - These calculations are "reasonable"
    - Don't rely too much on absolute numbers
- **Pummeling** – need to consider whole range of profiles/conditions borne by rail/wheel
- Roughness generally not a contributing factor re contact stress
- Wheel and rail (transverse) **profiles** control contact stress

Conclusions – cont'd

- Friction raises the stress levels (and damage) considerably
- The wheel nearly always slips on the rail
- **Stick and slip** regions in the contact patch
- **3rd body layer => negative friction** is a root cause of much noise, vibration, corrugation
- **Shakedown** is a useful approach for determining whether a wheel-rail contact is "good" or "bad"
- It is worth understanding and investing in contact mechanics to "get things right"

THANK YOU

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